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DYNAMIC GENERAL EQUILIBRIUM TAX MODELS WITH ADJUSTMENT COSTS

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This paper deals with some modifications in the dynamic structure of general equilibrium tax models.

Investment behavior is modelled by introducing adjustment costs. Investment is incorporated in an intertemporal equilibrium framework with perfect foresight. Short-run macroeconomic analysis, long-run growth theory, transition paths, and structural adjustment are integrated through microeconomic principles of intertemporal rationality.

The solution technique is based on linearization in continuous time. These ideas are illustrated in an aggregated model with the introduction of a consumption tax to replace a tax on capital income. Numerical simulations suggest that the modelling of forward-looking behavior and adjustment costs in general equilibrium tax models is important. This approach provides a more realistic basis for estimating the short- and medium-run effects, incidence effects, and welfare effects of fiscal policy.

Key words: Tax Models, Intertemporal Equilibrium, Adjustment Costs, Linearization.

1. Introduction: Dynamic general equilibrium tax models

This paper focuses on the dynamic formulation of general equilibrium tax models. General equilibrium analysis of taxation started with Harberger's static model (Harberger 1962). Harberger (1962) did not explicitly model time, and adjustment can be considered to occur instantaneously. More recent tax models are more disaggregated, and they are richer in institutional detail. For analyses within the tax tradition of applied general equilibrium analysis, see Shoven (1976), Keller (1980), Fullerton et al. (1981, 1983) and Shoven and Whalley (1984). Most of these models still carry over Harberger's basic theoretical structure. They abstract from imperfect competition, money, imperfect factor-mobility, uncertainty, and rationing.

Despite these potential weaknesses, empirical general equilibrium models are used to evaluate proposals for tax reform. They are especially useful in examining medium-run consequences in terms of distribution, efficiency, allocation and growth. Because of their emphasis on relative prices and their potential for detailed disaggregation, the models are useful to investigate the distributional consequences of policy. The focus on the interactive nature of the economic process renders them one of the most appropriate tools to study the medium-run consequences of public policy in a decentralized economic system. In addition, one can use them as a pedagogical

device to illustrate the operation of an economic system in which decisions are implemented on a decentralized level. Because one explicitly specifies the micro behavior of agents based on optimization, one can fruitfully perform welfare analysis as well as study the interaction of public policy and micro behavior.

Some attempts have been made to incorporate dynamics by using a recursive sequence equilibrium approach (see, e.g., Fullerton et al. (1981, 1983) and Bovenberg and Keller (1981)). These models allow for the growth of the capital stock over time. In a recursive fashion, one finds a sequence of temporary flow equilibria. This recursive technique is feasible because of the absence of forward-looking expectations. The economic agents are myopic. They do not use any information on the future in implementing their current decisions. Each temporary equilibrium is directly affected by previous equilibria only (Fig. 1).

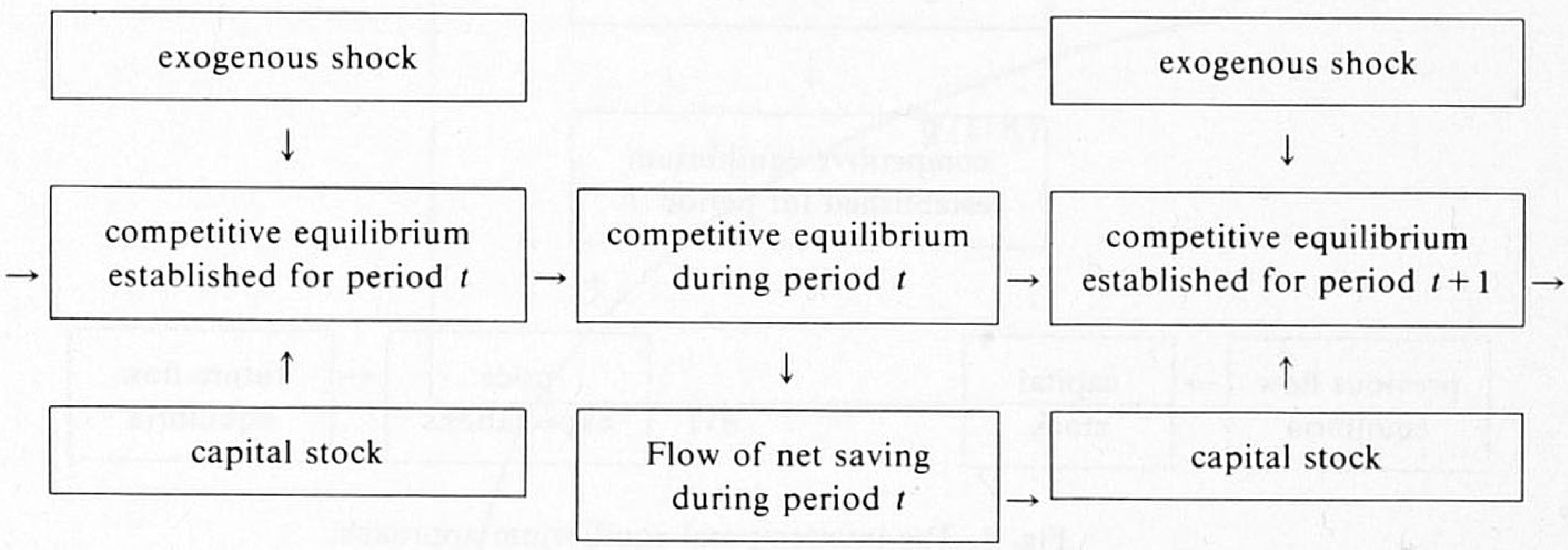

Fig. 1. The recursive sequence equilibrium approach.

In Section 2 we introduce the notion of intertemporal equilibrium. Here we specify forward-looking expectations. In Section 3 we discuss a second new element in the dynamic formulation of the tax models: adjustment costs in the installation of new capital equipment. Section 4 describes a solution technique for these intertemporal equilibrium models with adjustment costs, using linearization techniques in continuous time. We illustrate our general approach with a simple example in the class of fully dynamic models considered here. Section 4 includes the equations for this aggregated model with only one household sector and one production sector. In Section 5, we perform some numerical simulations for the introduction of a consumption tax. The results suggest the potential importance of the modelling of forward-looking behavior and of inertia in industry investment. This provides a more realistic modelling of structural adjustment over time. Section 6 contains the conclusions and summarizes some limitations of the approach presented.

2. Model formulation: Intertemporal equilibrium

For intertemporal equilibrium approaches similar to this paper, see Hall (1971), Brock and Turnovsky (1981), Sachs (1982), and Abel and Blanchard (1983). Our approach is characterized by two main features. First, decentralized behavior is derived from intertemporal optimization. The production sector maximizes the present value of its after-tax cash flow. Households optimize intertemporal utility over an infinite horizon. Thus, saving behavior is derived from standard microeconomic principles. This is different from arbitrarily specified saving behavior in Feldstein (1974, 1975), Boadway (1979) and Bernheim (1981).

The second feature of the intertemporal equilibrium approach is perfect foresight. Current decisions are based on the future path of prices that will actually unfold over time (barring future, unanticipated shocks in exogenous variables). Each period's transactions are directly affected by future equilibria (Fig. 2). It is not feasible to solve recursively through time.

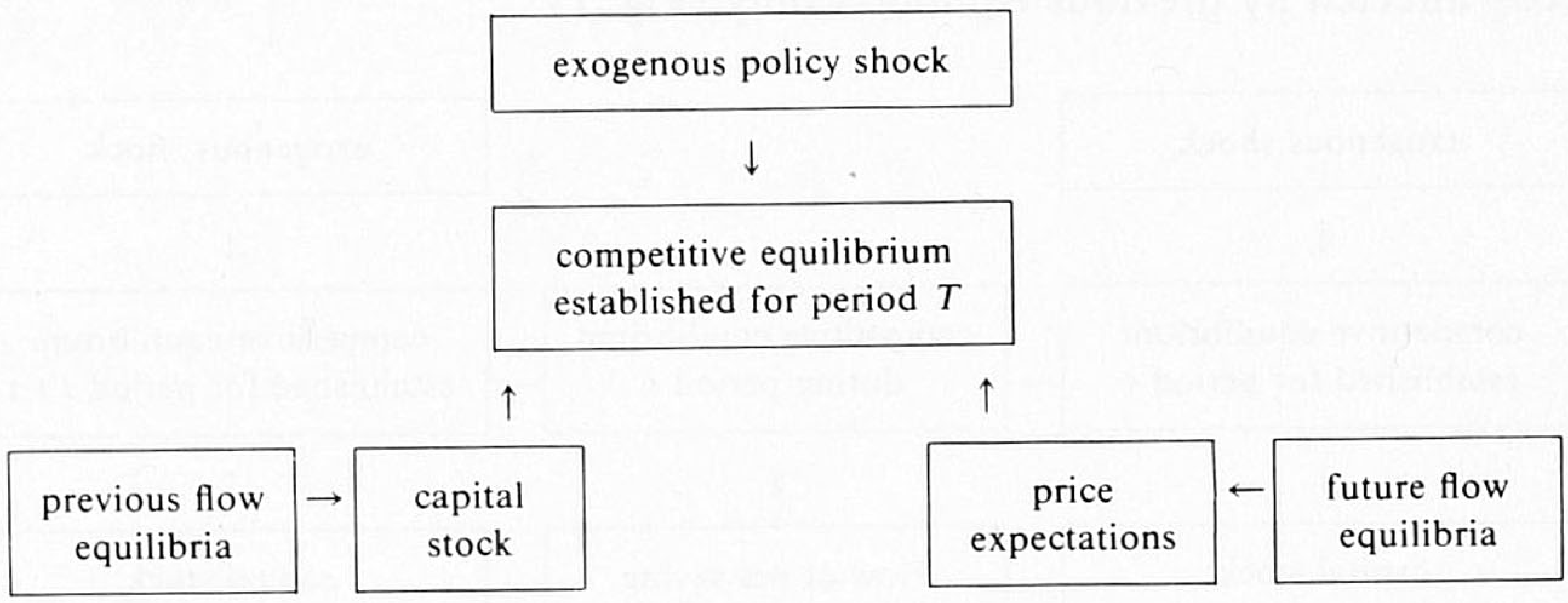

Fig. 2. The intertemporal equilibrium approach.

An intertemporal equilibrium approach can be viewed as the dynamic equivalent of Walrasian general equilibrium theory. With price expectations consistent with the model, and microeconomic behavior derived from dynamic optimization, one measures the dynamic welfare effects by substituting the solutions into the intertemporal utility functions.

3. Model formulation: Investment and adjustment costs

In the framework presented here, we derive the transition path to a new balanced growth equilibrium. Thus, the model formulation differs both from a comparative static formulation as in Harberger (1962), Shoven (1976) and Keller (1980), and from a comparative steady-state formulation as in Feldstein (1975) and Summers 1981a). In this paper, adjustment occurs gradually over time. This adjustment is slowed down by the specification of an installation function. This function captures absorptive capacity constraints in the investment process by modelling rising marginal adjustment costs in installation. The adjustment costs approach to investment is developed by Lucas (1967) and Treadway (1969). The installation function describes how the endowment of capital services, K, and the flow of gross investment,

I, combine to give $\dot{K}$, the rate of change in the endowment of capital. That is:

$$\dot{K} = G(I, K). \tag{1}$$

It is assumed that the installation function $G(\cdot\,,\cdot)$ is homogeneous of degree one in its arguments. Thus, we can rewrite (1) by dividing through by K as

$$\dot{K}/K = G(I/K, 1) = g(I/K), \quad g' > 0, g'' < 0. \tag{2}$$

The marginal productivity of capital goods declines with the rate of gross investment. The faster one expands the stock of capital services, the more capital goods per additional unit of capital services one needs. See Fig. 3.

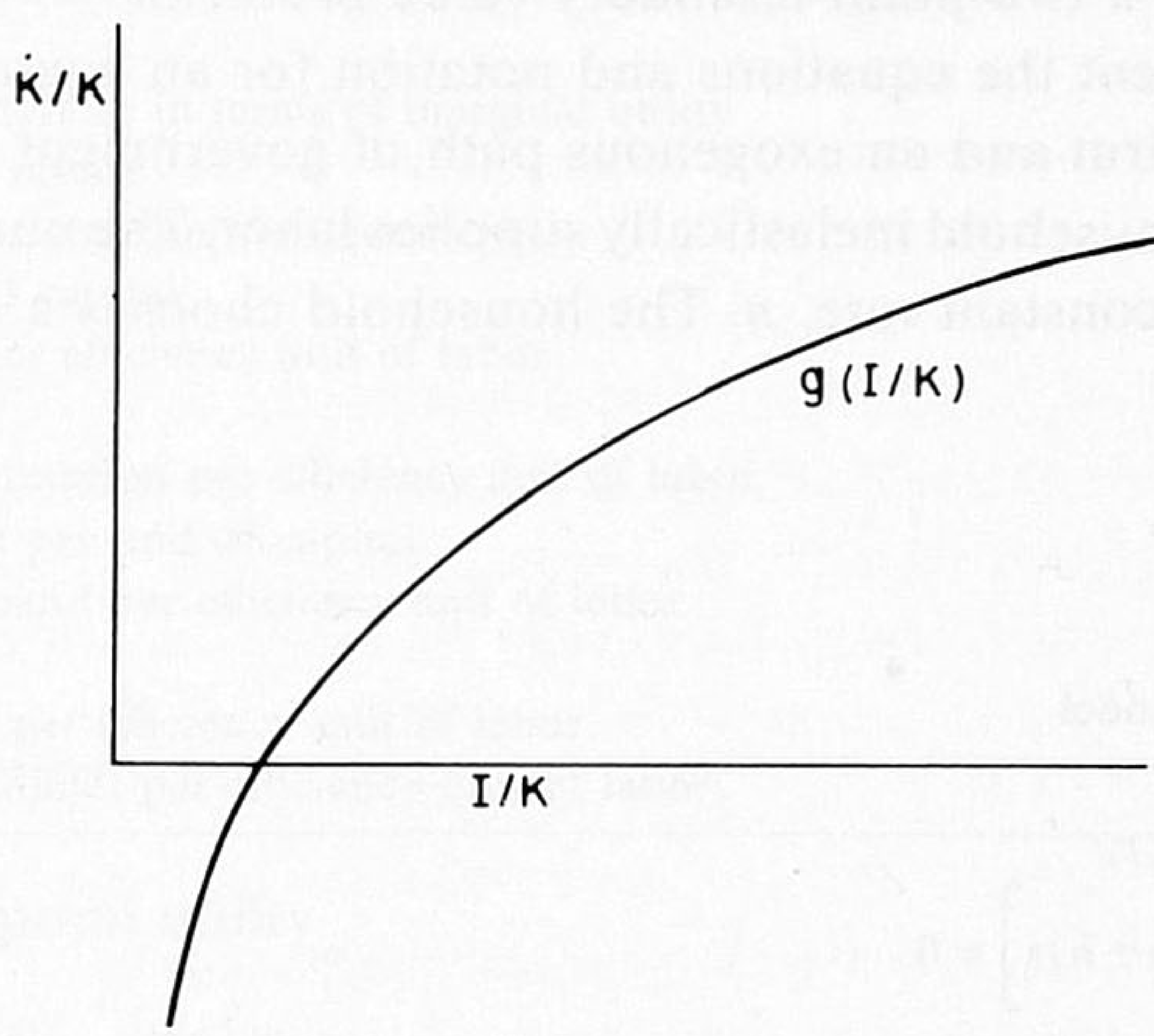

Fig. 3. The installation function.

The production sector maximizing its present value subject to this installation technology gives rise to Q-theory type investment functions (Hayashi (1982)). Q, representing the ratio of the financial market value of capital and the reproduction cost of capital, was introduced by Tobin (1969).

Q-theory links the real sector with the financial sector. The financial assets introduced can be interpreted as equity claims on physical captial. Asset valuation is derived from forward-looking behavior of rational investors—despite the absence of money. For a treatment of money within an intertemporal equilibrium framework, see Brock and Turnovsky (1981) and Sachs (1982).

By introducing independent industry investment, we alter the 'macro-closure' of general equilibrium tax models.[1] Investment in physical capital and household saving is no longer identically equal. A financial sector intermediates between decentralized households, storing their wealth in financial assets, and decentralized firms requiring financial captial to finance their investments in physical capital.

The empirical relationship between Tobin's Q and gross investment can be used to estimate the installation technology (see, e.g., Summers (1981b) and Hayashi (1982)).

[1] See Sen (1963) for a discussion of the closure issue.

4. Model solution: Linearization and illustration

In order to solve for an intertemporal equilibrium, we use the necessary conditions for decentralized dynamic optimization. For each agent we formulate an hamiltonian with state variables, costate variables and control variables. For a similar approach see Abel and Blanchard (1983). Dynamic optimization provides the canonical equations in the form of differential equations for state variables and costate variables, initial conditions for state variables and terminal conditions (the transversality conditions) for costate variables. In mathematical terms the intertemporal equilibrium system is a two-point boundary value problem.

In Table 1 we present the equations and notation for an aggregated model with one household, one firm and an exogenous path of government expenditures. The infinitely long-lived household inelastically supplies labor. The number of household members grows at a constant rate, n. The household chooses a consumption path

Table 1

The general equilibrium model

$$\dot{v}_{kt} = (r_t - n - h)v_{kt} + w_t - c_t p_t \tag{3}$$

$$\lim_{t\to\infty} v_{kt} \exp\left[-\int_0^t r_v \,\mathrm{d}v + (n+h)t\right] = 0 \tag{4}$$

$$\rho_t p_t = u'(c_t) \tag{5}$$

$$\frac{\dot{\rho}_t}{\rho_t} = b^x + (1-f)h - r_t \tag{6}$$

$$\lim_{t\to\infty} \rho_t v_{kt} \exp[-(b^x - n - fh)t] = 0 \tag{7}$$

$$\frac{\dot{k}_t}{k_t} = g(x_t) - n - h \tag{8}$$

$$w_t = p_t(f(k_t) - k_t f'(k_t))t_{Lt}t_{ct} \tag{9}$$

$$q_t g'(x_t) = p_t(t_{ct}/t_{It}) \tag{10}$$

$$\frac{\dot{q}_t}{q_t} = r_t - \frac{t_{ct}t_{kt}p_t f'(k_t)}{q_t} - g(x_t) + \frac{t_{ct}p_t x_t}{t_{It}q_t} \tag{11}$$

$$\lim_{t\to\infty} q_t k_t \exp\left[-\int_0^t r_v \,\mathrm{d}v + (n+h)t\right] = 0 \tag{12}$$

$$v_{kt} = q_t k_t \tag{13}$$

$$k_0 = \hat{k}_0 \tag{14}$$

$$g_t = (f(k_t) - k_t x_t)(1 - t_{ct}) + t_{ct}[(1 - t_{kt})k_t f'(k_t) + ((1 - t_{It})/t_{It})k_t x_t + (1 - t_{Lt})(f(k_t) - k_t f'(k_t))] \tag{15}$$

$$c_t + k_t x_t + g_t = f(k_t) \tag{16}$$

$$r_t = b^x + (1-f)h \tag{17}$$

$$\rho_0 = 1 \tag{18}$$

Endogenous: c_t, ρ_t, k_t, w_t, x_t, q_t, v_{kt}, t_{ct}, p_t and r_t;

Exogenous: g_t, $\hat{k}_0$, t_{Lt}, t_{kt} and t_{It};

The first derivative of a function m_t with respect to time, $\mathrm{d}m/\mathrm{d}t$, is denoted by $\dot{m}_t$.

The first derivative of a function $h(\cdot)$ with respect to its argument u, $\mathrm{d}h/\mathrm{d}u$, is denoted by $h'(\cdot)$.

Table 1 (continued)

Variable definitions:

Parameters:

b^x	pure rate of time preference
n	rate of population growth
h	rate of labor-augmenting technological change
f	degree of homogeneity of the felicity function $u(\cdot)$
tax ratios:	
t_L	ratio of supply price and demand price for labor
t_k	ratio of after-tax capital income and before-tax capital income
t_I	ratio of supply price and demand price for capital goods
t_c	ratio of supply price and demand price for consumption goods
spot prices:	
ρ	price of the numeraire in terms of marginal utility
r	after-tax rate of return
p	demand price for consumption goods
q	price of installed capital
w	after-tax wage per efficiency unit of labor
flows:	
c	household consumption per efficiency unit of labor
x	gross investment per unit of capital
g	government demand per efficiency unit of labor
stocks:	
v_k	financial wealth per efficiency unit of labor
k	endowment of capital per efficiency unit of labor

such that intertemporal utility

$$\int_0^\infty u(c_t\,\mathrm{e}^{ht})\,\mathrm{e}^{-b^x t}\,\mathrm{e}^{nt}\,\mathrm{d}t \tag{19}$$

is maximized. Utility is separable over time. The felicity function $u(.)$ is assumed to be homothetic in its argument. The wealth constraint in differential equation is represented by (3) and (4). Applying optimal control theory to the household maximization problem, we arrive at conditions (5), (6) and (7).

Optimizing the hamiltonian for the firm with respect to the control variables labor demand and investment demand, we derive (9) and (10) respectively. $f(k)$ stands for the production function relating output per efficiency unit of labor to capital services per efficiency unit of labor, k. (11) and (12) describe the optimal path for the costate variable q. The equality between 'average' q and 'marginal' q (see Hayashi (1982)) is formalized by (13). With exogenous government demand, the tax on consumption adjusts endogenously to balance the public budget. A balanced public budget is given by (15). (16) establishes equilibrium in the goods market. The numeraire is defined by (17) and (18).

One can solve the intertemporal equilibrium problem with non-linear numerical methods such as the multiple shooting technique. These solution methods are applied by Lipton et al. (1982), Bruno and Sachs (1982) and Summers (1981b). Instead, we adopt a dynamic generalization of the Johansen linearization technique.[2]

[2] The linearization technique enables us to derive some analytic results for the aggregated model. In contrast to the numerical simulations in Feldstein (1974), Fullerton et al. (1981, 1983) and Summers (1981b), we can explicitly examine how the results are related to the various structural parameters involved.

This linearization method is familiar from static computable general equilibrium models. See, e.g., Johansen (1964), Keller (1980), Dixon et al. (1981) and Bovenberg and Keller (1984). In these static models, one takes the existence of an initial equilibrium as given. One log-linearizes the model equations around this initial equilibrium and one computes the relative changes in this static equilibrium following a policy shock. By assuming that the various elasticities are constant at their initial equilibrium values, these solutions hold exactly only for infinitesimal changes. However, one can use them as a linear approximation for larger changes.

Here we adapt the linearization technique for comparative dynamics as follows. The existence of an initial steady-state equilibrium is taken as given. In a dynamic framework, one not only log-linearizes the static equations, but one also log-linearizes the differential equations around the initial steady state. For similar approaches, see Chamley (1981) and Judd (1983). In this way one finds the relative changes from the initial balanced growth path following an unanticipated change in policy.

From the linearized static equations, one solves for the static endogenous variables in terms of the exogenous variables and the dynamic endogenous variables. These static reduced form expressions for the static endogenous variables may be interpreted as temporary equilibrium solutions. In order to solve for a temporary equilibrium, one needs the information that is contained in the dynamic endogenous variables. Depending on the kind of information that these dynamic variables embody, one can distinguish between forward-looking dynamic endogenous variables and backward-looking dynamic endogenous variables. The forward-looking variables contain information on the future of the system. They instantaneously jump in response to new information in such a way that the terminal conditions are met. These variables correspond to the unstable roots of the system. The information on the history of the economy is embodied in the backward-looking variables. These correspond to the stable roots and to the initial conditions.[3]

Substituting the temporary equilibrium solutions in the linearized differential equations, we arrive at a linear dynamic system in continuous time in the dynamic endogenous variables and the exogenous variables only. The linearized model can be reduced to a two-dimensional saddle-point stable dynamic system. The backward-looking endogenous variable is the capital-labor ratio, k. As the forward-looking endogenous variable, it is convenient to choose the costate variable q.[4] Writing the two-dimensional column vector with the dynamic endogenous variables as $z = (k, q)$, and the column vector with exogenous variables as t, the linearized model can be formalized as

$$\dot{\bar{z}}_t = C\bar{z}_t + D\bar{t}_t, \tag{20}$$

$$\bar{k}_0 = 0 \tag{21}$$

[3] A recursive sequence equilibrium approach (discussed in Section 1) is characterized by the absence of forward-looking dynamic endogenous variables. When all the dynamic endogenous variables are backward-looking, one can solve recursively through time.

[4] Alternatively, one can use the investment variable, $x = I/K$, as the forward-looking endogenous variable.

where a barred variable represents a relative change. The 2×2 elasticity matrix C is presented in Table 2.

The two-dimensional system (20)–(21) can be solved analytically. With more disaggregated models, the linear dynamic systems are too large to be solved analytically. However, one can solve these models numerically. For numerical solution techniques in discrete time, see Blanchard and Kahn (1980). For solutions in continuous time, see Buiter (1984).

Table 2

The elasticities matrices in the initial steady state

The matrix C in eq. (20):

$$b\begin{bmatrix} \dfrac{a^I}{a^c A_1}(\sigma_c + A_2(1+y)) & \dfrac{y}{A_1} \\ (1+y)\dfrac{(1-\alpha_k)}{\sigma_k} + \dfrac{\sigma_x a^I}{y a^c A_1}(\sigma_c + A_2(1+y)) & \dfrac{\sigma_x}{A_1} \end{bmatrix}$$

The vector D in eq. (20):

$$b\begin{bmatrix} \dfrac{a^I}{a^c A_1}(1+y) \\ \dfrac{\sigma_x a^I(1+y)}{y a^c A_1} - (1+y) \end{bmatrix}$$

For notational convenience, we drop the time subscripts denoting the initial balanced growth path.

Variable definitions:

$a^I = \dfrac{xk}{f(k)}$ investment share in output

$a^c = \dfrac{c}{f(k)}$ consumption share in output

$\alpha_k = \dfrac{kf'(k)}{f(k)}$ capital share in production

$\sigma_k = -(1-\alpha_k)\dfrac{kf''(k)}{f'(k)}$ substitution elasticity between capital and labor in production

$\sigma_c = -\dfrac{u''(c)c}{u'(c)}$ Elasticity of marginal felicity (the inverse is the intertemporal substitution elasticity)

$\sigma_x = -\dfrac{g''(x)x}{g'(x)}$ Elasticity of marginal productivity of investment in installation

The second derivative of a function $h(\cdot)$ with respect to its argument u, $\mathrm{d}^2h/\mathrm{d}u^2$, is denoted by $h''(\cdot)$.

$$b = b^x - n - fh, \qquad y = \frac{g'(x)x}{b}, \qquad y = \frac{xg'(x)}{b},$$

$$A_1 = \frac{a^I}{a^c}\sigma_c + \sigma_x, \qquad A_2 = \frac{(\sigma_c - 1)(1-t_k) + ((1-\alpha_k)/\sigma_k)(t_L - t_k)}{t_k}$$

One can establish the following steady-state relations

$$a^I = \alpha_k \frac{t_k y}{(1+y)}, \qquad \frac{a^I}{a^c} = \frac{\alpha_k y t_k}{t_L + \alpha_k(t_k - t_L) + y(1-\alpha_k)t_L} = \frac{kx}{c}$$

5. An application: The consumption tax

The intertemporal equilibrium model may now be applied to a typical tax policy issue. We analyze the dynamic effects of the introduction of a tax on consumption to substitute for a tax on capital income.

If the tax on capital income is the only exogenous policy instrument considered, we can reduce the vector of exogenous variables in equation (20) to a scalar. Table 2 contains the corresponding two-dimensional column vector D. We model an unanticipated permanent decrease in the tax rate on capital income.[5] The consumption tax adjusts to balance the public budget.

It is claimed that the tax reform analyzed here will yield substantial welfare gains by reducing the intertemporal distortion in the tax system. Furthermore, it is argued that these welfare gains will 'trickle down' to the wage earners. In the long run, capital deepening will cause real wages to rise, and capital owners will not benefit greatly from the reduction of the tax rate on their income.

These claims are supported by Summers' (1981a) analysis. Here we test these results in a somewhat different framework. Our model differs from Summers' formulation in two major ways.[6]

First, Summers' results are steady-state results. We explicitly compute the transition path to a new steady state. Second, we specify rising marginal adjustment costs in installation. The Summers model does not include this feature.

In order to focus on these two major differences in model formulation, the parameter values for the numerical simulations are kept as close as possible to Summers (1981a). See Table 3. We also carry out some sensitivity analysis with respect to the elasticities of marginal felicity (determining the saving elasticity) and the elasticity of the marginal productivity of capital goods in installation (determining the investment elasticity).

Assuming infinitely elastic investment, corresponding to no adjustment costs in installation, case I represents Summers (1981a).[7] In case II, we assume infinitely elastic saving behavior.[8] The unitary elasticity in installation is taken from Bruno and Sachs (1982). By combining inelastic saving behavior from case I with inelastic investment behavior from case II, we arrive at case III. Here we model rigidities in both saving and the installation of new capital equipment.

[5] The intertemporal equilibrium approach with a solution technique based on linearization allows for the modelling of arbitrarily complex changes in current and future policy. See Judd (1983). For an elaborate discussion of the analytic results corresponding to the policy experiment performed here, see Bovenberg (1984).

[6] In addition to these two major differences, the models of household behavior differ. Summers (1981a) specifies an overlapping generations model without bequests. We assume infinitely long-lived households. Furthermore, in contrast to Summers, our solutions are based on linearization. Consequently, the solutions hold exactly for infinitesimal changes in policy only.

[7] As in Summers (1981a), felicity is logarithmic. Recall, however, the differences in the modelling of the planning horizon and bequests.

[8] This approximates Summers (1981b). He fixes the after-tax real rate of return. Lacking an explicit model of household behavior with an intertemporal utility function, he does not deal with welfare effects. Summers' (1981b) implicit estimate for the elasticity of the marginal productivity of investment is 2.2.

Table 3
Parameter values

Case		I	II	III
b^x	pure rate of time preference	0.03	*	*
n	population growth	0.015	*	*
h	productivity growth	0.02	*	*
f	homogeneity felicity	0	*	*
$g'(x)x$	value of gross investment relative to value of capital stock	0.035	*	*
α_k	capital share	0.25	*	*
t_k	tax rate on capital income	0.5	*	*
t_L	tax rate on labor income	0.2	*	*
σ_k	substitution elasticity in production	1	*	*
σ_c	intertemporal substitution elasticity in consumption	1	0	1
σ_x	elasticity of marginal productivity investment	0	1	1

* Denotes the same numerical value as in case I.

Table 4 contains the numerical results. Although in the first case the adjustment is rather rapid, the actual welfare gains are substantially smaller than indicated by the steady-state calculations of Summers (1981a). See Fig. 4.[9] The welfare gains are only 64% of the steady-state efficiency effect. The modelling of foresight tends to reduce the welfare gains. Savers anticipate future declines in the return on capital. This anticipation reduces their saving and slows down the adjustment to a new balanced growth path with more efficient production. See also Auerbach and Kotlikoff (1983) and Ballard and Goulder (1982).

Turning to the distributional effects, it can be seen that in the new steady state real wages rise relative to real profits. Discounting the earnings of labor and capital over the entire transition, however, human wealth (labor earnings) declines relative

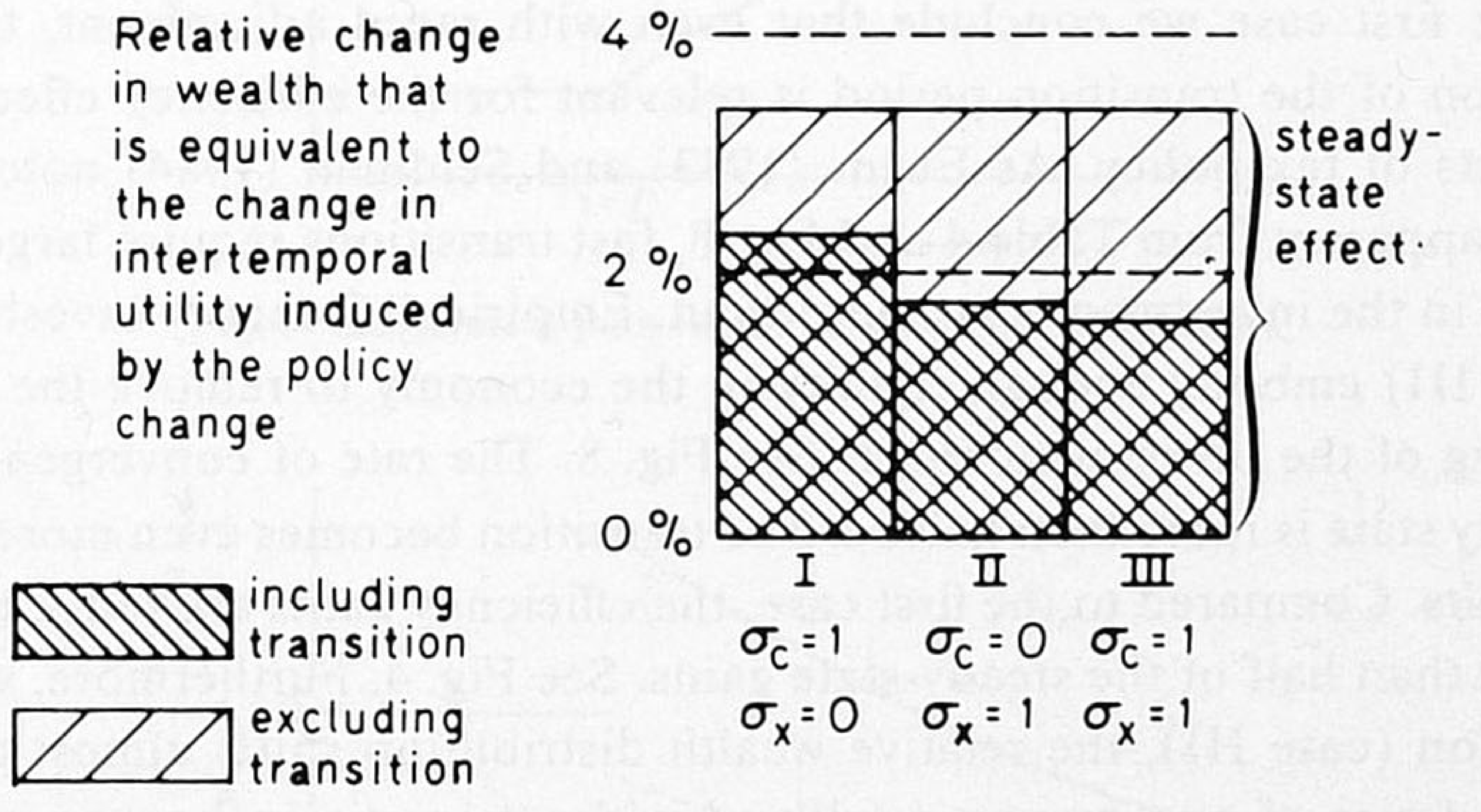

Fig. 4. Reduction of the capital income tax from 50% to 45% and efficiency effects (linear approximation).

[9] Welfare gains are measured as the compensating variation. I.e., we compute the relative change in initial wealth that is equivalent to the change of intertemporal utility induced by the unanticipated policy change.

Table 4
Numerical results for an unanticipated permanent decrease of the tax on capital income

Case	I	II	III
Marginal results (Exact for infinitesimal changes only)			
annual rate of convergence to new steady state (%)	6.6	3.1	2.6
ratio of instantaneous and long-run change in the share of investment in production	2.50	1.18	1.00
ratio of discounted efficiency gains including and excluding the transition	0.64	0.52	0.49
ratio of distributional effects including and excluding the transition	−1.06	−2.64	−3.03
Linear approximations for a decrease of the tax on capital income of 5 percentage points			
Relative change in the share of investment in production at time of policy shock (%)	25.0	11.8	10.0
Efficiency effect (%)	2.1	1.8	1.7
Distributional effect (%)	−3.5	−8.7	−10.0

Notes:
Efficiency effects are measured as the compensating variation.
Distributional effects are measured as the relative change in the ratio of labor earnings and capital earnings.

to non-human wealth (capital earnings). Thus, the steady-state equity effect is reversed. See Fig. 5.

From the first case we conclude that even with rapid adjustment, the explicit consideration of the transition period is relevant for the efficiency effects and the equity effects of tax policy. As Evans (1983) and Seidman (1984) note, however, and is also apparent from Table 4 and Fig. 8, fast transitions require large short-run movements in the investment share in output. Empirical Q-theory investment functions (case III) embody enough rigidity in the economy to remove the unrealistic overshooting of the investment share. See Fig. 8. The rate of convergence towards a new steady state is more than halved. The transition becomes even more important for the results. Compared to the first case, the efficiency gains are reduced by about 25% to less than half of the steady-state gains. See Fig. 4. Furthermore, with inertia in installation (case III), the relative wealth distribution shifts almost three times as much in favor of non-human wealth as without inertia in investment (case I). See Fig. 5.

Clearly, adjustment costs in installation affect the welfare effects and the distributional effects of policy. Comparing the third case with the first two, the relative importance of these adjustment costs for the rate of convergence to a new balanced

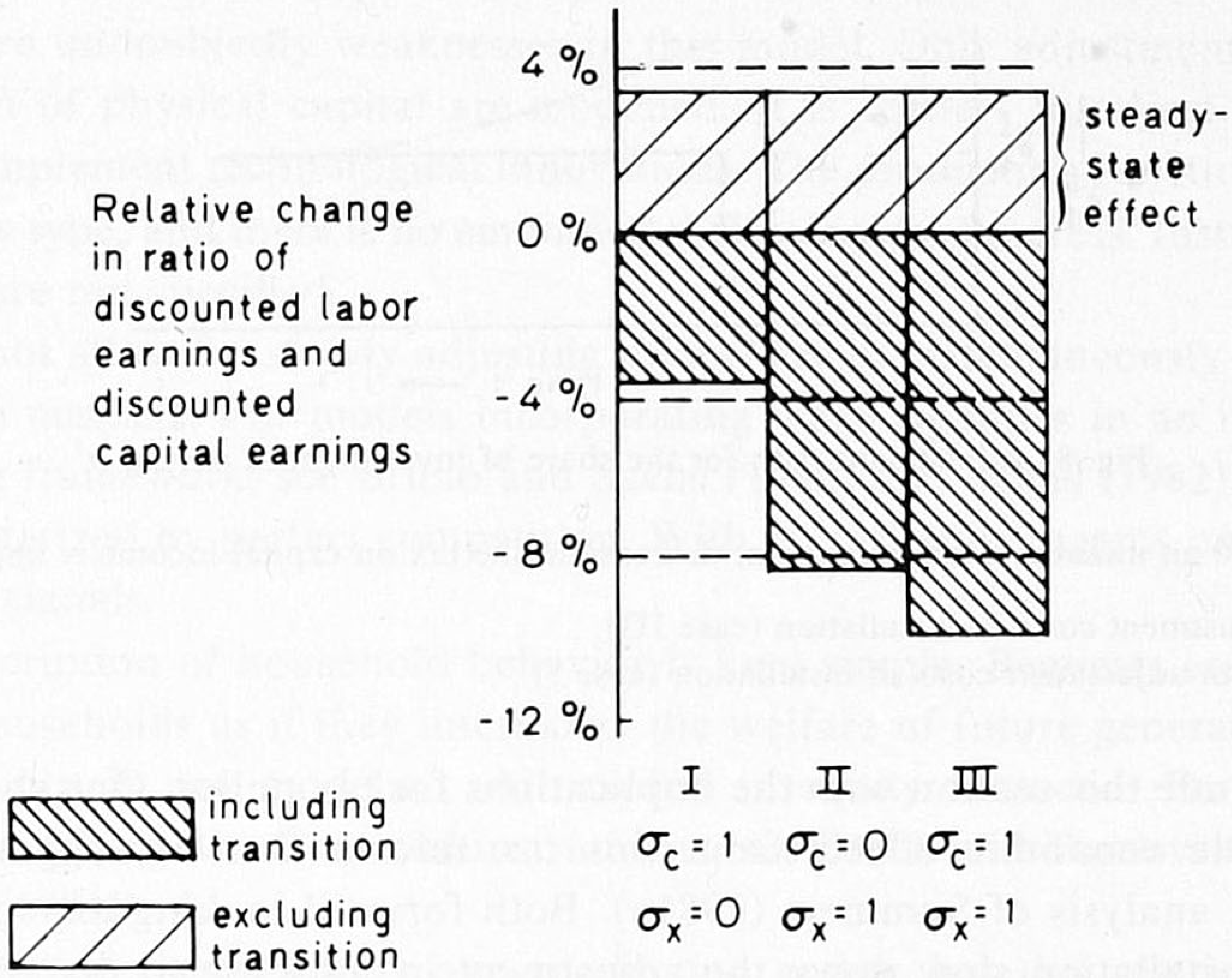

Fig. 5. Reduction of the capital income tax from 50% to 45% and distributional effects (linear approximation).

growth path becomes clear. Case III demonstrates the importance of rigidities in both installation and the intertemporal allocation of consumption. By ignoring inertia in installation (case I), we speed up the transition more substantially than by ignoring inertia in the intertemporal allocation of household consumption (case II). In Figs 6-8, we graph the transition paths for key variables with and without adjustment costs in the installation of new capital equipment.

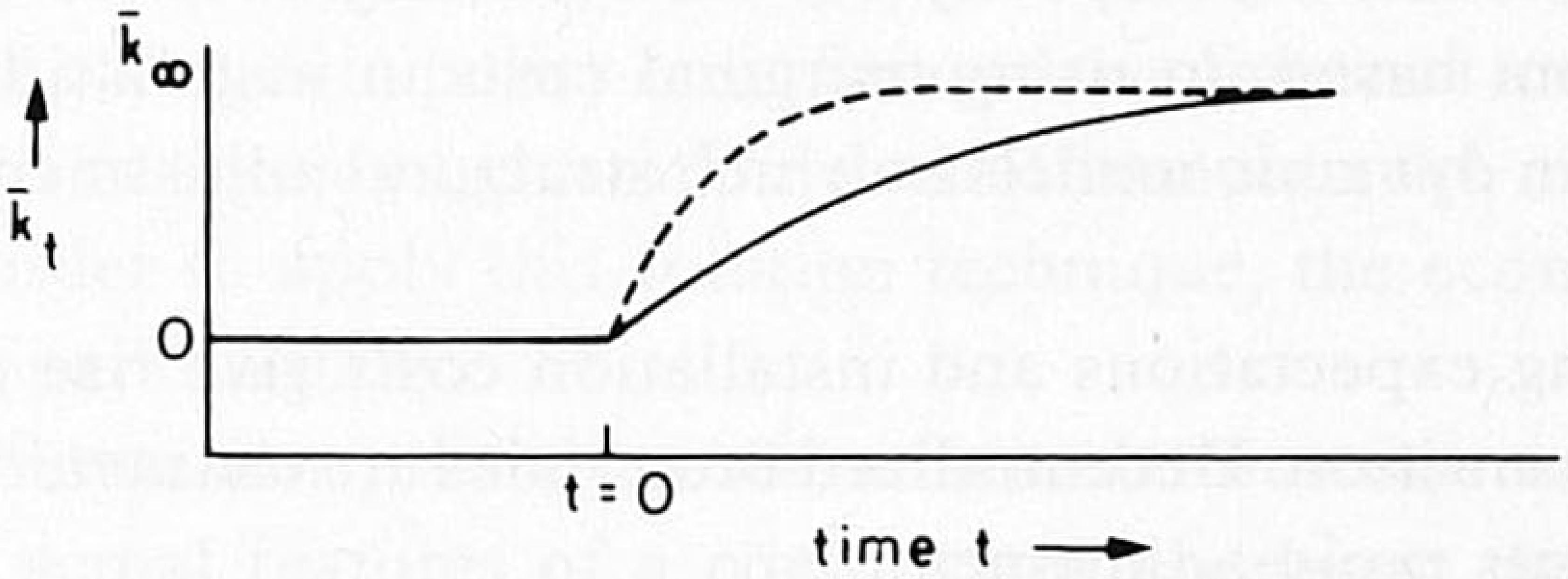

Fig. 6. Adjustment path for the endowment of capital k.

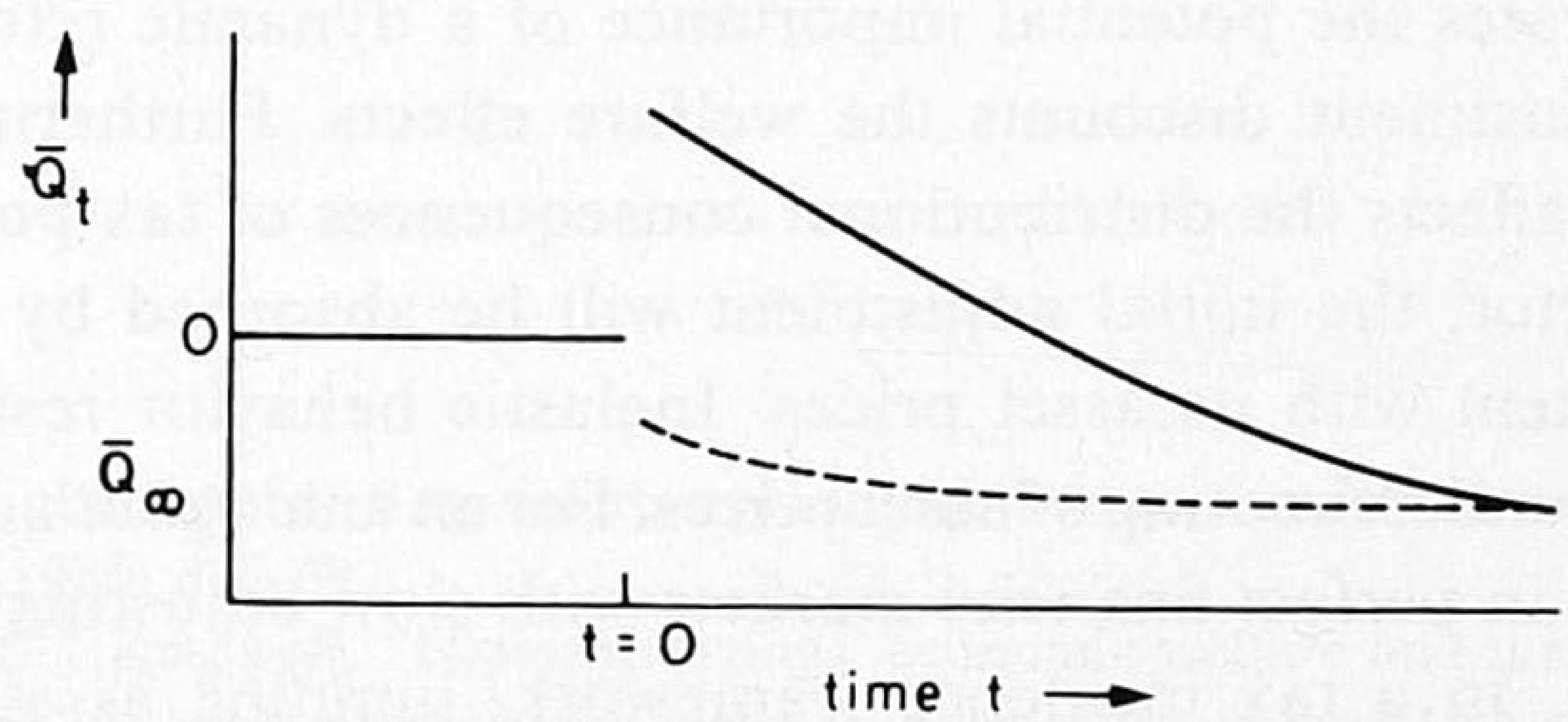

Fig. 7. Adjustment path for the real value of capital $Q = q/p$.

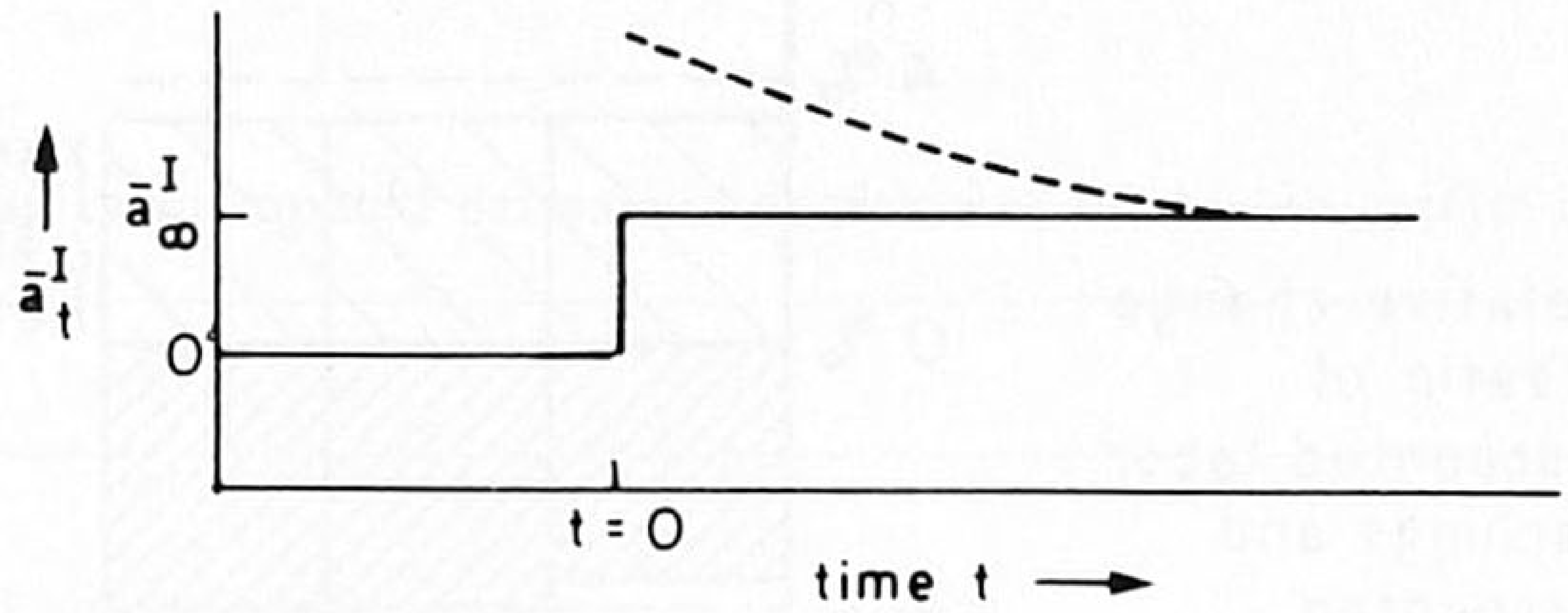

Fig. 8. Adjustment path for the share of investment in output a^I.

At time $t = 0$ an unanticipated permanent decrease in the tax on capital income is implemented.

—— with adjustment costs in installation (case III)

- - - - - without adjustment costs in installation (case I)

We conclude this section with the implications for tax policy. One should expect less dramatic economic effects from this tax reform than is suggested by the steady-state analysis of Summers (1981a). Both forward-looking expectations and inertia in installation slow down the adjustment in response to tax reform. Once one recognizes the slower adjustment, it becomes more difficult to reform the tax system in a Pareto-welfare improving manner. 'Economy-wide' welfare gains are reduced. In addition, these smaller gains tend to accrue to capital owners rather than to workers.

6. Conclusions

This paper proposes some reformulations in the dynamic structure of general equilibrium tax models. We specify forward-looking behavior and independent industry investment based on rising marginal costs in installation. These elements are implemented in dynamic models in which structural adjustment occurs gradually over time.

Forward-looking expectations and installation costs give rise to a more realistic modelling of the transition. Decentralized economies are characterized by substantial inertia that prevents rapid adjustment.

An aggregated intertemporal equilibrium model with adjustment costs is applied to the introduction of a consumption tax to replace a tax on captial income. This application illustrates the potential importance of a dynamic reformulation of tax models. Slow adjustment discounts the welfare effects. Furthermore, the way an economy adjusts affects the distributional consequences of tax policy. With a slow adjusting real sector, the initial adjustment will be absorbed by the dual system, i.e., the price system with its asset prices. Inelastic behavior results in successive overshooting and undershooting of asset prices. For an analogous adjustment process of exchange rates in perfect financial markets with slow adjusting real markets, see Dornbush (1976). In a tax incidence framework, jumping asset prices represent windfall gains and windfall losses. These have important distributional effects.

There are undoubtedly weaknesses in this model. Only adjustment costs in the installation of physical capital are modelled. It is costless to adjust capital-labor ratios or implement technological innovations. The production functions are of the putty-putty type, and there is no embodied technological progress. Installation costs for labor are not specified.

We do not allow for slowly adjusting prices. Prices instantaneously clear current and future markets. For models incorporating wage rigidities in an intertemporal optimizing framework, see Bruno and Sachs (1982) and Sachs (1982). All markets are characterized by perfect competition. With no rationing, agents only use prices as market signals.

The description of household behavior is kept simple. Bequests are handled by treating households as if they internalize the welfare of future generations.

Uncertainty is not modelled. All capital is equity financed. For approaches allowing for alternative financial instruments, see Brock and Turnovsky (1981) and Summers (1981b).

This paper deals with only one production sector, but this restriction may be relaxed. Bovenberg (1984) specifies a Harberger-type model with a corporate sector and a noncorporate sector. With an installation technology for each industry, physical capital is imperfectly mobile among the two industries. Differential capitalization effects and the gradual reallocation of capital between sectors are modelled. In principle, the approach proposed can be implemented in more disaggregated applied tax models with several households and several industries. Bovenberg (1984) applies these ideas to a two-country model with one household and one industry in each country.

This paper employs a solution technique based on linearization in continuous time. The limitations of this method are twofold. First, the solutions are 'marginal' and hold for small changes in policy only. Second, we linearize around a steady-state equilibrium which is taken as given. We do not compute the initial intertemporal equilibrium. In order to apply this solution technique, the economy must be on a balanced growth path initially.

Despite these limitations, the dynamic reformulations represent a step forward to include more actual features of a complicated economic world in tax models. This provides a more realistic description of the dynamics of a decentralized economic system, and therefore, a more useful tool for fiscal policy evaluation.

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